

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT3

Branch: Diploma (All)

Semester : 2

Date : 25/04/2018

Time : 10:30 To 01:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

- a) $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = \underline{\hspace{2cm}}$
 (A) ax^{n-1} (B) nx^{n-1} (C) na^{n-1} (D) None of these
- b) $\lim_{x \rightarrow 0} \frac{\sin 3x}{2x} = \underline{\hspace{2cm}}$
 (A) $3/2$ (B) $2/3$ (C) $1/3$ (D) $1/2$
- c) $\lim_{x \rightarrow 0} \left(1 - \frac{2x}{3}\right)^{\frac{4}{x}} = \underline{\hspace{2cm}}$
 (A) e (B) $e^{-8/3}$ (C) $e^{2/3}$ (D) $e^{-3/2}$
- d) $\frac{d(\sec x)}{dx} = \underline{\hspace{2cm}}$
 (A) $\sec x \tan x$ (B) $-\sec x \tan x$ (C) $\cos e x \cot x$ (D) $-\cos e x \cot x$
- e) $\frac{d(x^2 + 2x + 7)}{dx} = \underline{\hspace{2cm}}$
 (A) $2x$ (B) $2x + 1$ (C) $2x + 2$ (D) None of these
- f) $\frac{d(\tan^{-1} x)}{dx} = \underline{\hspace{2cm}}$
 (A) $\frac{-1}{\sqrt{1-x^2}}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{1}{1+x^2}$ (D) $\frac{-1}{1+x^2}$
- g) $\frac{d\left(\frac{1}{x}\right)}{dx} = \underline{\hspace{2cm}}$
 (A) $\log x$ (B) $-\frac{1}{x}$ (C) $\frac{1}{x^2}$ (D) $-\frac{1}{x^2}$
- h) $\int e^x dx = \underline{\hspace{2cm}}$



(A) $\log x + c$ (B) $e^x + c$ (C) 1 (D) 0

i) $\int \frac{1}{\sqrt{x^2 + a^2}} dx = \underline{\hspace{2cm}}$

(A) $\log \left| x + \sqrt{x^2 + a^2} \right| + c$ (B) $\log \left| x + \sqrt{x^2 - a^2} \right| + c$ (C) $\frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$

(D) $\frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$

j) $\int_0^1 \frac{2}{1+x^2} dx = \underline{\hspace{2cm}}$

(A) π (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) None of these

k) $-3(5, -1, -2) + 8(1, 1, 0) = \underline{\hspace{2cm}}$

(A) (7, 11, 6) (B) (7, -11, 6) (C) (7, 11, -6) (D) (-7, 11, 6)

l) If θ is the angle between the vectors \vec{x} and \vec{y} then $\cos \theta = \underline{\hspace{2cm}}$.

(A) $\frac{|\vec{x} \times \vec{y}|}{|\vec{x}||\vec{y}|}$ (B) $\frac{\vec{x} \times \vec{y}}{|\vec{x}||\vec{y}|}$ (C) $\frac{\vec{x} \cdot \vec{y}}{|\vec{x}||\vec{y}|}$ (D) $\frac{\vec{x} \times \vec{y}}{|\vec{x} \times \vec{y}|}$

m) If $x(3, 1) + y(4, 2) = (1, 0)$ then $x = \underline{\hspace{2cm}}$ and $y = \underline{\hspace{2cm}}$.

(A) 1, 2 (B) 2, 1 (C) 1, $\frac{1}{2}$ (D) 1, $-\frac{1}{2}$

n) $(1, 1, 1) \cdot (-1, 2, -3) = \underline{\hspace{2cm}}$.

(A) -2 (B) 2 (C) 1 (D) -1

Attempt any four questions from Q-2 to Q-8

Q-2

Attempt all questions

(14)

a) If $\vec{a} = 2i + j - k$, $\vec{b} = i - j + 2k$ and $\vec{c} = i - 2j + k$ then find the direction cosines of $\vec{a} + \vec{b} - 2\vec{c}$. **(5)**

b) Prove that angle between two vectors $i + j - k$ and $2i - 2j + k$ is $\sin^{-1} \left(\frac{\sqrt{26}}{3\sqrt{3}} \right)$. **(5)**

c) Evaluate: $\lim_{x \rightarrow 2} \frac{x^7 - 128}{x^4 - 16}$ **(4)**

Q-3

Attempt all questions

(14)

a) Prove that $\lim_{x \rightarrow 3} \frac{\sqrt{x+2} - \sqrt{5}}{\sqrt{x+4} - \sqrt{7}} = \frac{\sqrt{35}}{5}$. **(5)**

b) Evaluate: $\lim_{x \rightarrow \frac{\pi}{4}} \frac{2 - \sec^2 x}{1 - \tan x}$ **(5)**

c) If $y = \frac{\log x}{x}$ then find $\frac{dy}{dx}$ at $x = 1$. **(4)**

Q-4

Attempt all questions

(14)



a) $S = t^3 - 6t^2 + 8t - 4$ gives the distance travelled by a body in t seconds. Find velocity and acceleration at $t = 4$ second. (5)

b) Find $\frac{dy}{dx}$ if $y = \log \sqrt{\frac{a+x}{a-x}}$. (5)

c) Simplify: $(10i + 2j + 3k) \cdot [(i - 2j + 2k) \times (3i - 2j - 2k)]$. (4)

Q-5

Attempt all questions

(14)

a) Find unit vector which is perpendicular to $\vec{a} = i + j + k$ and $\vec{b} = 2i - 2j + k$. (5)

b) Find $\frac{dy}{dx}$ if $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$. (5)

c) Evaluate: $\lim_{x \rightarrow 0} \frac{3^{2x} - 2^{2x}}{x}$ (4)

Q-6

Attempt all questions

(14)

a) Forces $(1, 2, 3)$, $(-1, 2, 3)$ and $(-1, 2, -3)$ act on a particles and the particle moves from the point $(0, 1, -2)$ to $(-1, 3, 2)$. Find the work done by the forces. (5)

b) Prove that $\int_0^{\frac{\pi}{2}} \frac{\tan x}{\tan x + \cot x} dx = \frac{\pi}{4}$. (5)

c) Find derivative of $f(x) = e^x$ using definition. (4)

Q-7

Attempt all questions

(14)

a) Evaluate: $\int x \log x dx$ (5)

b) Find the area of the region bounded between curve $y = x^2$ and straight – line $x = 2$. (5)

c) For what value of R , vectors $2i - 3j + 5k$ and $Ri - 6j - 8k$ are perpendicular to each other? (4)

Q-8

Attempt all questions

(14)

a) If $f'(x) = 4x^2 + 6x - 3$ and $f(1) = 2$ then find function $f(x)$. (5)

b) Find $\frac{dy}{dx}$ if $y = (\sin x)^x$. (5)

c) Evaluate: $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx$ (4)

